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ASYMMETRIC LEARNING SPILLOVERS

Ву

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CES 93-7 April 1993

Abstract

In this paper, I employ a linear-quadratic model of an industry characterized by learning by doing to examine the implications of asymmetric learning spillovers. Importantly, I show that distribution of spillover benefits can influence market structure in ways that can not be seen in models where spillovers are symmetric. If spillovers are asymmetric, a tradeoff between improved industry performance and increased market concentration can arise which does not occur when they are symmetric. This tradeoff leads to a policy dilemma; whether to promote static or dynamic efficiency in markets where learning is important.

Keywords: learning by doing, spillovers, industry evolution.

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This paper is a revised version of a chapter from my Ph.D dissertation at the University of Oregon. I would like to thank Van Kolpin, Jim Adams, B. Kelly Eakin, Daniel McMillen, Sang Nguyen and Wesley Wilson for helpful comments. Any opinions, findings or conclusions expressed here are those of the author and do not necessarily reflect the views of the Census Bureau.

I. INTRODUCTION

Learning by doing is the process through which unit costs decrease as a firm accumulates production experience. Learning has important implications for firm behavior and market performance. It creates an intertemporal link between the strategies firms employ today and the competitive environment they find themselves in tomorrow. This link can be exploited by incumbent firms to achieve an absolute cost advantage and erect entry barriers (see Spence [1981] and Fudenberg and Tirole [1983]). The strategic effectiveness of the learning curve depends on the appropriability of experience. If entrants are able to learn from existing firms, incumbents will be less able to exploit the learning curve to deter entry (see Ghemawat and Spence [1985]).

This result, however, is derived from models in which it is invariably assumed that all firms benefit from rival experience equally. That is, learning spillovers are symmetric across firms. The possibility of asymmetric spillover learning is not addressed in the literature. This is regrettable since there is no reason to suppose that spillovers should be symmetric.

Geographical location, research and development expenditures and other idiosyncratic firm characteristics are likely to affect how individual firms learn from the experience of their rivals.

A model of learning by doing that incorporates spillover asymmetries may, therefore, approximate actual conditions in

industry more accurately. The purpose of this exercise is to investigate whether any of the major results from models where spillovers are symmetric differ when spillover asymmetries are present. It is especially important to know if the result that spillovers improve both the static and dynamic performance of industries is an artifact of the assumed distribution of spillover benefits.

I employ a linear-quadratic model of an industry characterized by learning by doing to examine the implications of asymmetric learning spillovers. I show that distribution of spillover benefits can influence market structure in ways that cannot be seen in models where spillovers are symmetric. When spillovers are asymmetric, a tradeoff between improved industry performance and increased market concentration can arise which does not occur when they are symmetric. This tradeoff leads to a policy dilemma; whether to promote static or dynamic efficiency in markets where learning is important. Finally, I also show that the possibility of learning asymmetries has implications for the specification of empirical models of learning.

In the next section, I briefly discuss the progress of research on learning by doing. I present the model and discuss some computational issues in section III. Results from the linear-quadratic simulations are given in section IV.

Conclusions are provided in section V.

II. BACKGROUND

Empirical evidence of a negative relationship between production costs and experience has existed for decades (see Wright [1936], Alchian [1963], Rapping [1965], and Sheshinski, [1967]). The phenomenon of learning by doing is now well accepted by researchers in fields as diverse as economics, engineering and management. In economics, Arrow [1962] was the first to examine the theoretical issues surrounding learning by doing. A decade later consulting firms were advising their corporate clients to exploit the learning curve aggressively in order to obtain cost advantages which would enable them to exercise market power (e.g., see Boston Consulting Group [1972]). By the mid 1980s, however, it was evident that the strategic effectiveness of the learning curve decreases if the benefits of the firm's experience spill over to its rivals and potential rivals (Ghemawat and Spence [1985] and Dasgupta and Stiglitz [1988]).

Lieberman [1982, 1984] provides empirical evidence which suggests that firms benefit more from rival experience than from their own. This result is interesting from a policy perspective because spillovers tend to both improve the dynamic performance of industries (by increasing the speed of cost reduction) and reduce market concentration. That is, spillovers promote both static and dynamic economic efficiency. These findings have received considerable attention and have reduced interest in further research, both theoretical and empirical, on learning by

doing. However, these findings are also based on models which employ overly restrictive assumptions about the distribution of learning benefits across firms.

III. MODEL

Competition in an industry characterized by learning by doing can be modelled as a dynamic game. Assume that there are n firms and T discrete time periods, where T may be finite or infinite. At the beginning of each period, firms choose quantities of a homogeneous output, q_{it} . Firm i's costs in period t, $C_{it}(q_{it},X_t)$, are a function of current output and the cumulative experience vector, X_t . In this paper, I index experience with cumulative production so that for each firm i, $x_{it} = \frac{t}{s=1}q_{is}$ and $X_t = (x_{it})_{i=1}^n$. The objective of each firm is to choose values of q_{it} to maximize

$$\Pi_{i} = \sum_{t=1}^{T} \delta_{i} \{ P(\boldsymbol{q}_{t}) \, \boldsymbol{q}_{it} - C_{it}(\boldsymbol{q}_{it}, X_{t}) \}$$
 (1)

$$s.t. X_t - X_{t-1} + Q_{t-1} (2)$$

$$X_0 - 0$$

where $q_t = \prod_{i=1}^n q_{it}$, $Q_{t-1} = (q_{it-1})_{i=1}^n$ and $P(q_t)$ is the industry inverse demand function. The \prod_i term is a discount factor.

Quantity is the only choice variable in the model. Although strategic variables such as research and development expenditures

and firm location are likely to affect learning by doing, I abstract from these in the current model. In effect, I assume that decisions regarding such variables are sunk and that their impact on learning is constant over the time horizon being modeled. This helps to keep the problem manageable.

Linear-Quadratic Specification

Solving the model in this general form is not feasible. So to examine the implications of asymmetric learning spillovers, I use a linear-quadratic approximation for which solution algorithms exist. To obtain the linear-quadratic specification additional structure must be placed on the model. First, assume that the inverse demand function is linear and given by $P_t = a - bq_t$. Next, firm i's cost function is defined as $C_{it} = (c_{i0} - ('_iX_t)q_{it}) + ('_iX_t)q_{it}$ when $(c_{i0} - ('_iX_t) > c_{iT}) + (c_{iT})q_{it}$ otherwise. In this definition, c_{i0} and c_{iT} are firm i's initial and terminal unit costs, respectively and $('_i = ((c_{i1}, (c_{i2}, \ldots, (c_{in})))))$ is the vector of learning parameters. These measure the effectiveness with which firm i exploits firm j's experience to reduce its unit costs. As previously discussed, the spillover parameters, $(c_{ij}, (c_{im}, (c_{im})))$ are likely to differ across firms (i.e., $(c_{ij}, (c_{ij}, (c_{im}, (c_{im})))$).

For the simulations below assume there are 5 firms which operate over a 15 period time horizon (T=15). Further, assume that firm 1 enters in the $1^{\rm st}$ period and the other four firms all enter in the $3^{\rm rd}$ period. This pattern of entry reflects the situation in an industry for a new product where an innovator

maintains a monopoly for a short time. Learning by doing is most likely an important factor in the rivalry between producers in new industries or in industries where there has been a recent innovation. Also, assume that each firm uses an identical discount factor, $*_i$ = 0.9. The demand parameters, a and b, are given values of 91/30 and 7/300, respectively. Finally, assume that the initial and terminal unit cost for each firm are c_{i0} = 1.0 and c_{iT} = 0.7, respectively. These are the same demand and cost parameters employed in Ross [1986] and they conveniently imply that the surplus maximizing output in each period is 100.2

Kydland's [1975] algorithm calls for rewriting the maximization problem in equations (1) and (2) as

$$v_{it} = \max_{q_{it}} \{ \mathbf{p}'_{it} \mathbf{z}_t + \mathbf{z}'_t \mathbf{\theta}_{it} \mathbf{z}_t + \mathbf{\delta}_i \mathbf{v}_{it:1} \}$$
(3)

$$s.t. z_t - Az_{t-1} + Bq_t (4)$$

$$z_0 - 0$$

Equation (3) is firm i's value function at time t. The 2n vector $z'_t = (x_{1t}, q_{1t}, \dots, x_{nt}, q_{nt})$ is the augmented state vector. The 2n vector p_{it} and the 2nx2n matrix $\mathbf{1}_{it}$ contain the model's demand and cost parameters. The matrices A and B are constructed of zeros and ones such that (3) and (4) conform to (1) and (2). Once the outcome in the last period is specified (i.e., once v_{it} is computed) it is possible to solve the model recursively.

Solutions can be computed assuming either Cournot or von

Stackelberg behavior. In the simulations below, I assume firms

move simultaneously as in the Cournot case.

Computational Issues

Before discussing the main results of the paper, I want to briefly discuss a potential problem of using the preceding framework to examine learning by doing. In economic applications, endpoint conditions must make economic sense. In the case of learning by doing, this requires that the evolution of unit costs from c_{i0} to c_{iT} result from the firm accumulating enough experience (either own or rival). In the present specification, the terminal unit costs are such that once all firms have reached the bottom of their learning curves the game reverts to the symmetric Cournot-Nash solution until the end of the time horizon. Therefore, as long as each firm reaches the bottom of its learning curve by accumulating sufficient experience, the finite linear-quadratic model is an exact representation of the infinite model with the same parameters.

Ross [1986] used this recursive solution procedure to model learning by doing without ensuring that the endpoint conditions made economic sense. I computed solutions under both 10 (as in Ross) and 15 period time horizons. In both cases, I assume firms benefit from their own experience only (i.e., $(i_{ij} = 0)$ for all j...i) where $(i_{ij} = 0.0015)$. Table I lists summary statistics for the two simulations. Reported in the table are either the date

at which the firm exhausts its learning possibilities (t_i^*) or, if it does not, the unit cost it attains by the 9^{th} period. Also listed are the discounted profits for each firm for the first 10 periods.

In the 10 period solution, only firm 1 is able to reach the bottom of its learning curve before the last period. All firms reach the bottom of their learning curves by the last period in the 15 period case. Figure 1 illustrates how divergent the two solutions are by displaying the time path of outputs for one of the identical entrants from the 3rd to 9th periods. The figure shows that when the algorithm is correctly applied, the entrants produce much more during the early periods. In fact, they endure losses in the period subsequent to entry in order to accumulate experience more quickly. In the 10 period case, however, the entrants do not place enough weight on investing in experience. This is because the model artificially imposes the terminal unit cost for each of the entrants by the tenth period regardless of whether they have accumulated enough experience or not.

IV. RESULTS

In this section, I report and compare the solutions to the linear-quadratic model under three spillover alternatives. I compare solutions in cases where spillovers accrue both symmetrically and asymmetrically to the case with no spillover learning. These comparisons focus on how the spillover alternatives affect industry performance and market

concentration. Previous authors have studied the entry deterring effects of learning by doing. In the present model, however, entry is exogenous. Thus, I examine concentration to see how spillovers affect market structure. Finally, I examine the effects of asymmetric spillovers on the behavior of price-cost margins. The behavior of price-cost margins over time has implications for the specification of certain econometric models of learning by doing.

Three Spillover Scenarios

Table II reports, in addition to the statistics given in table I, total discounted surplus (profits plus consumer surplus) for the model under the three different learning scenarios. In the first column, I report the results from the case where learning benefits are completely proprietary (no spillovers). In the second and third columns of table II are the results from the symmetric and asymmetric spillover cases, respectively. The values of the learning parameters, in each case, are reported at the foot of the table.

In all 3 cases, firm 1 reaps benefits from incumbency. This is seen in table II by comparing post-entry profits of the incumbent to those of the entrants. In each case, the incumbent makes higher post-entry profits than its rivals. This is due to the cost advantage the incumbent acquires before the other firms enter the market. The incumbent is able to achieve this advantage, in each case, because the spillover parameters are

smaller in magnitude than the proprietary learning parameters (i.e., spillovers are not complete). The learning curve would confer no benefits to incumbents if spillovers were complete and symmetric (i.e., if experience was a pure public good, which in the present model implies (i,j) for all i,j because they would be unable to gain any cost advantage by accumulating experience.

Comparing the first two columns of table II shows that symmetric spillovers dramatically increase the speed of learning over the no spillover case. This improves firm profits and total surplus. With symmetric spillovers the incumbent invests less in experience during the early periods. This results from two causes. First, the incumbent reduces output rates in the early periods to prevent the entrants from learning from his experience. Second, the incumbent also benefits from the experience of his rivals and can, therefore, achieve a given amount of cost reduction with a smaller investment in own experience.

In the asymmetric case, assume that only firm 2 benefits from rival experience. Expectedly, the results in table II show that firm 2 reaches the bottom of its learning curve sooner and earns higher profits than under the proprietary case. The other firms reduce their output rates in the early periods to restrict the benefits firm 2 can gain from their experience. This keeps prices high in the early periods resulting in larger profits for the industry as compared with the base case. Total surplus is

also greater under the asymmetric case than under the base case.

Figure 2 shows how market concentration evolves over the time horizon for each of the learning scenarios. The measure of concentration used is the Herfindahl index. In each case the Herfindahl index evolves to the symmetric equilibrium value of 1/n. As compared with the base case, market concentration decreases under symmetric spillovers and increases under the asymmetric spillover case. Interestingly, the increased concentration in the asymmetric case results from firm 2 producing less and free riding on the experience of its rivals. That is, the firm with the advantage in spillover learning earns higher profits by settling for a smaller market share.

Combined, the results in table II and figure 2 show that when spillovers are asymmetric a tradeoff between market performance and concentration can arise. That is, asymmetric spillovers can imply a tradeoff between static and dynamic economic efficiency. This important possibility is not revealed by models where spillovers benefits are symmetric across firms. The distribution of spillover benefits across firms affects market structure and performance. Policies intended to promote static efficiency by reducing market concentration may impair the dynamic performance of industries where learning by doing is important.⁸

Behavior of Price-Cost Margins

The final result of this paper concerns the behavior of

price-cost margins under different spillover assumptions.

Previous theoretical models (see Spence [1981] and Lieberman [1982]) show that, when experience is a pure public good and the elasticity of demand is constant over time, unit costs and prices decline at approximately the same rate. This result has been exploited to justify using price as a proxy for cost in learning regressions (Lieberman [1982], [1984]).

Table III lists the price-cost margins resulting from three spillover alternatives over time. The last two columns list the margins from the symmetric and spillover cases discussed above and the first column lists them for a case where spillovers are both complete and symmetric (i.e., $\binom{1}{ij} = \binom{1}{ij}$ for all i,j). Although the linear-quadratic case precludes a constant elasticity of demand over time, the price-cost margins in the table when spillovers are complete and symmetric are fairly stable. The asymmetric case in the third column of table III shows the most variation in the margins over time.

This suggests that if spillovers are asymmetric, price-cost margins will not remain constant over time and price would not be an acceptable proxy for unit costs. In this case, if no cost data are available the econometrician needs to model the process that generated the observed market prices in order to make meaningful inferences about learning parameters.

V. CONCLUSIONS

The treatment of asymmetric spillovers is an important

omission in the theoretical literature on learning by doing. Studies where spillover benefits accrue to firms symmetrically show that spillovers tend to improve industry performance and reduce market concentration. It is unlikely that firms in actual industries benefit from rival experience equally. A number of firm specific characteristics such as research and development expenditures, location and so on are likely to affect the extent to which any given firm is able to learn from the experience of its rivals.

In this paper, I show that the result that spillovers reduce concentration and improve performance is dependent upon the assumed distribution of spillover benefits. When spillovers are asymmetric, I found that a tradeoff between improved performance and increased market concentration can arise. To the extent that market regulators base their decisions on structural characteristics (e.g., market concentration), they run the risk of impairing the performance of industries where learning is important. When learning is an important feature of competition in an industry, researchers and policy makers need to be aware of the pattern of learning benefits across firms in the industry.

I also discussed the effects of asymmetric learning spillovers on the behavior of price-cost margins over time. This result has implications for certain econometric specifications of learning by doing. In particular, specifications where

spillovers are required to be symmetric are flawed, if spillovers in the industry being examined are actually asymmetric.

TABLE I.
Summary Statistics for Simulations under two
Alternative Time Horizons

	T = 10	T = 15	
t* ₁ t* ₋₁	8 -	10 15	
C ₋₁₉	0.868	-	
$\begin{matrix} \textbf{A}_1 \\ \textbf{A}_1 \mathring{} \\ \textbf{A}_{-1} \end{matrix}$	123.73 37.86 17.19	100.44 14.62 7.42	

 $\overline{A_{\text{l}}}$ discounted post-entry profits for firm 1.

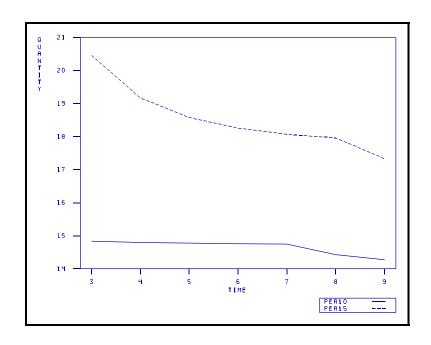


FIGURE 1.
Time Path of Output for Entrants under 10 and 15 Period Time Horizons.

TABLE II.
Summary Statistics for Simulations under three Spillover Alternatives

	Proprietary [†]	Symme	tric [‡]	Asymmetric§
t* ₁	10	7	10	
t* ₁ t* ₂ t* _{3,4,5}	15 15	10 10	6 15	
A ₁ A ₁ , A ₂ A _{3,4,5}	109.43 23.61 15.61 15.61	124.93 38.34 30.71 30.71	112.91 27.28 26.15 18.99	
	747.48 015 and $\binom{1}{1} = 0.0$ 015 and $\binom{1}{1} = 0.0$ 015 for all i, and 0.0 for all i2	003 for all i, and $(_{2j} = 0.0009)$	j ij.	2,

 $[\]boldsymbol{A}_{\text{l}^{\wedge}}$ Discounted post-entry profits for firm 1.

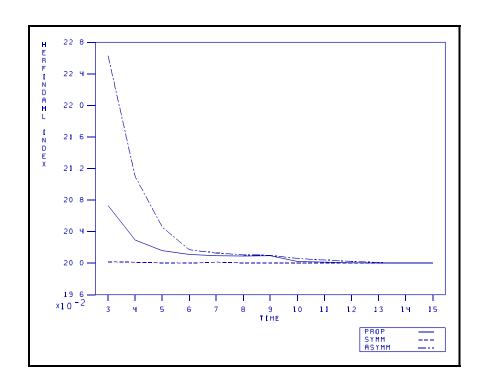


FIGURE 2. Herfindahl Indices for Three Spillover Alternatives.

TABLE III.
Price-Costs Margins for
Three Spillover Alternatives

t	Com&Symm	Symmetric	Asymmetric
3	0.338	0.225	0.045
4	0.347	0.253	0.115
5	0.356	0.282	0.165
6	0.366	0.311	0.201
7	0.376	0.336	0.231
8	0.387	0.361	0.260
9	0.389	0.388	0.287
10	-	0.389	0.307
11	-	_	0.327
12	-	_	0.347
13	-	_	0.368
14	-	_	0.389
15			<u> </u>

APPENDIX

To solve the model, first posit that the value function in equation (3) takes the quadratic form

$$V_{it} = V_{it} + r'_{it}z_{t-1} + z'_{t-1}S_{it}z_{t-1}$$
 (A1)

Plugging into (3) gives

$$v_{it} = \max_{q_{it}} \{ (p'_{it} + \delta_i r'_{it\cdot 1}) z_t + z'_t (\Theta_{it} + \delta_i S_{it\cdot 1}) z_t + \delta_i v_{it\cdot 1} \}$$
(A2)

which has necessary conditions for a Nash equilibrium

$$b'_{i}(p_{it} + \delta_{i}r_{it:1}) + b'_{i}(\theta_{it} + \theta'_{it} + \delta_{i}(S_{it:1} + S'_{it:1}))z_{t} = 0$$
A3)

where b_i is the i^{th} column of B. The n expressions given by (A3) can be stacked and rewritten as

$$k_t + H_t Z_t - k_t + H_t (\mathbf{A} Z_{t-1} + \mathbf{B} \mathbf{q}_t)$$
 (A4)

and solving for \boldsymbol{q}_t provides the solution

$$\boldsymbol{q}_{t} - \boldsymbol{d}_{t} + E_{t} \boldsymbol{z}_{t-1} \tag{A5}$$

where d_t and E_t have dimensions nx1 and nx2n, respectively. Finally $<_{it}$, r_{it} and S_{it} are determined recursively by (A6)

,

$$\vee_{it} = \delta_{i} \vee_{it \cdot 1^{+}} [\; (p'_{it} \cdot \delta_{i} r'_{it \cdot 1}) \cdot d'_{t} B' (\theta_{it} \cdot \theta'_{it} \cdot \delta_{i} (S_{it \cdot 1} \cdot S'_{it \cdot 1}) \;) \;] \; (A \cdot B E_{t})$$

(A7)

$$r'_{it} = [p'_{it} \cdot \delta_{i} r'_{it\cdot 1} \cdot d'_{t} B' (\Theta_{it} \cdot \Theta'_{it} \cdot \delta_{i} (S_{it\cdot 1} \cdot S'_{it\cdot 1}))] (A \cdot BE_{t})$$

and

$$S_{it} - (\mathbf{A} + \mathbf{B}E_t)'(\mathbf{\Theta}_{it} + \mathbf{\delta}_i S_{it-1}) (\mathbf{A} \cdot \mathbf{B}E_t)$$
 (A8)

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ENDNOTES

- 1. The algorithm I employ is due to Kydland (1975, 1977) and has been applied by Reynolds (1986) and Ross (1986).
- 2. Spence (1981) and Fudenberg and Tirole (1983) have shown that a social planner seeking to maximize total surplus will set price in each period equal to the terminal value of unit costs.
- 3. See the appendix for more details on the recursive computation of the equilibrium solutions.
- 4. I employ a larger value of the own learning parameters than Ross to help ensure that firms reach the bottom of their learning curves by the last period.
- 5. The results in the first column do not match Ross's results exactly as I allow a positive discount rate and employ a different value of the own learning parameter. However, this does not qualitatively alter the results.
- 6. While I make this assumption for ease of exposition, it is possible to conceive of such a situation occurring in practice. For example, it might be that only firm 2 employs resources to reverse engineer the products of its rivals. The decision to devote such resources is made outside the context of the present model and is viewed as a sunk cost.
- 7. The Herfindahl index is computed as the sum of squared market shares.
- 8. I also analyzed the model where the incumbent owned the advantage in spillover learning. In this case, profits were higher for each firm than under the proprietary case. However, total surplus decreased as compared to the no spillover case. This was due to decreased output by the incumbent because it could free ride on the experience of the entrants. There was no tradeoff as the asymmetric spillovers increased market concentration and decreased market performance.